Controlled Dynamics of Interfaces in a Vibrated Granular Layer

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We present experimental study of a topological excitation, *interface*, in a vertically vibrated layer of granular material. We show that these interfaces, separating regions of granular material oscillation with opposite phases, can be shifted and controlled by a very small amount of an additional subharmonic signal, mixed with the harmonic driving signal. The speed and the direction of interface motion depends sensitively on the phase and the amplitude of the subharmonic driving.

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Despite their ubiquity and many practical applications, an understanding of the fundamental dynamical behavior of granular materials remains a serious challenge [1]. One of the main obstacles for the development of a continuous description of granular flow is the difficulty in performing quantitative experiments under controlled conditions. Testing of theoretical models of the basic excitations of the system is especially important. It has been shown recently [2–5] that thin layers of granular materials subjected to vertical vibration exhibit a diversity of patterns which may play the role of such fundamental excitations. The particular pattern is determined by the interplay between driving frequency f and the acceleration amplitude $\Gamma = 4\pi^2 \mathcal{A} f^2/g$ of the cell, where $\mathcal{A}$ is the amplitude of oscillation and g is the acceleration due to gravity. Periodic patterns, such as squares and stripes, or localized oscillons vibrating with frequency $f/2$ appear at $\Gamma \approx 2.4$ [2–5]. At higher acceleration ($\Gamma > 3.72$), stripes and squares become unstable, and hexagons appear instead. Further increase of acceleration replaces hexagons with a non-periodic structure of interfaces separating large domains of flat layers oscillating with opposite phase with frequency $f/2$. These interfaces were called kinks in Ref. [3,4,6]. These interfaces are either smooth or "decorated" by periodic undulations depending on parameters [4,6]. For $\Gamma > 5.7$, various quarter-harmonic patterns emerge. Several theoretical approaches including molecular dynamics simulations, order parameter equations and hydrodynamic-type models, have been proposed recently to describe this phenomenology, see e.g. [7,8].

In this Letter we present an experimental study of the dynamics of interfaces in a vibrated thin layer of granular material. We find that an additional subharmonic driving results in a controlled displacement of the interface. In the absence of subharmonic driving, the interface drifts toward the middle of the cell. When the subharmonic frequency $f/2$ is slightly detuned, the interface moves periodically about the middle of the cell. The present experimental results are in agreement with theoretical predictions [9].

Interfaces in a granular layer separate regions of granular material oscillating with opposite phases with respect to the bottom plate of the vibrating cell. These two phases are related to the period-doubling character of the flat layer motion at large plate acceleration. Since an interface separates two stable symmetric dynamic phases, it can be interpreted as a topological defect, similar to a domain wall in ferromagnets separating regions of opposite magnetization [10]. Interfaces can only disappear at the walls of the cell or annihilate with other interfaces. The existence of the interfaces can be understood from the following consideration. Since grains in a layer lose their kinetic energy in multiple inter-collisions during landing at the plate, the behavior of a granular layer can be compared with the dynamics of a fully inelastic ball bouncing on a plate vibrating with amplitude $\mathcal{A}$ and frequency f [3,11]. In this case, for driving with acceleration Γ less than $\Gamma_0 \approx 3.72$ the ball lifts to the same height at each cycle [3,12]. Above Γ_0 the motion exhibits period-doubling, i.e. the heights of elevation alternate at each cycle. As a result, for $\Gamma > \Gamma_0$, the two states of the bouncing ball, differing by the initial phase, would coexist. If the analogous states of the bouncing flat layer co-exist in different parts of the cell, they have to be separated by an interface. These interfaces, found experimentally [3,4,6] and theoretically [9], are flat for high frequency drives and show transverse instability leading to periodic decoration at lower frequencies (see Fig. 1). In an infinite system a straight interface is immobile due to the symmetry between alternating states: the motion of flat layers on both sides of the interface is identical with a phase shift π. Additional driving at the subharmonic frequency $f_1 = f/2$ will break the symmetry between domains with opposite phases. Depending on the phase of the additional driving, Φ, with respect to the phase of the primary driving, the relative velocity of the layer and the plate at collision will differ on different sides of the interface, and the material on one side will be lifted to

a larger height than on the other side. As a result, the direction of the interface motion can be controlled by the phase Φ. The speed at a given Φ is determined by the amplitude of the subharmonic acceleration γ.

In our previous work we have developed a phenomenological description of the pattern formation in thin layers of granular material [7,9]. On the basis of our order parameter model we have predicted [9] that for small values of γ, the interface velocity $V = dX/dt$, X being the position of the interface, is a periodic function of Φ,

$$\frac{dX}{dt} = V_0 \sin(\Phi - \Phi_0) \qquad (1)$$

where $V_0 = \alpha\gamma$, and susceptibility α and phase shift Φ_0 are functions of the driving amplitude A and frequency f which can be estimated using the model Ref. [9].

We performed experiments with a thin layer of granular material subjected to two-frequency driving. Our experimental setup is similar to that of [2–4]. We used 0.15mm diameter bronze balls, and the layer thickness in most of our experiments was 10 particles. We performed experiments in a circular cell of 15.3 cm diameter and in a rectangular cell of 4×12 cm^2, which were evacuated to 2 mTorr. We varied the acceleration Γ and the frequency f of the primary driving signal as well as acceleration γ, frequency f_1 and the phase Φ of the secondary (additional) driving signal. The interface position and vertical accelerations were acquired using a CCD camera and accelerometers and further analyzed on a Pentium computer. The lock-in technique was used to measure the accelerations at f and $f/2$ frequencies in real time.

In the absence of additional driving the interface drifts toward the middle of the cell (see Fig. 1). We attribute this effect to the feedback between the oscillating granular layer and the plate vibrations due to the finite ratio of the mass of the granular material to the mass of the vibrating plate. Even in the absence of subharmonic drive, the vibrating cell can acquire a subharmonic motion from the periodic impacts of the granular layer on the bottom plate at half the driving frequency. If the interface is located in the middle of the cell, the masses of material on both sides of the interface are equal, and due to the anti-phase character of the layer motion on both sides, an additional subharmonic driving force is not produced. The displacement of the interface X from the center of cell leads to a mass difference Δm on opposite sides of the interface which in turn causes an additional subharmonic driving proportional to Δm. In a rectangular cell $\Delta m \propto X$. Our experiments show that the interface moves in such a way to decrease the subharmonic response, and the feedback provides an additional term $-X/\tau$ in the r.h.s. of Eq. (1), yielding

$$\frac{dX}{dt} = -X/\tau + V_0 \sin(\Phi - \Phi_0) \qquad (2)$$

The relaxation time constant τ depends on the mass ratio (this also holds for the circular cell if X is small compared to the cell radius). Thus, in the absence of an additional subharmonic drive ($\gamma = 0$), the interface will eventually divide the cell into two regions of equal area (see Fig. 1).

In order to verify this model we performed the following experiment. We positioned the interface off center by applying an additional subharmonic drive. Then we turned off the subharmonic drive and immediately measured the amplitude μ of the plate acceleration at the subharmonic frequency [13]. The results are presented in Fig. 2. The subharmonic acceleration of the cell decreases exponentially as the interface propagates to the center of the cell. The measured relaxation time τ of the subharmonic acceleration decreases with the mass ratio of the granular layer and of the cell with all other parameters fixed. The mass of the granular layer was varied by using two different cell sizes while keeping the thickness of the layer unchanged. For the cells shown in Fig. 1, we found that the relaxation time τ in the rectangular cell is about 4 times greater than for the circular cell (see Fig. 2). This is consistent with the ratio of the total masses of granular material (52 grams in rectangular cell and 198 grams in circular one). In a separate experiment the mass of the cell was changed by attaching an additional weight of 250 grams to the moving shaft, which weights 2300 grams. This led to an increase of the corresponding relaxation times of 15-25 %. The relaxation time τ increases rapidly with Γ (see Fig. 2, inset).

When an additional subharmonic driving is applied, the interface is displaced from the middle of the cell. For small amplitude of the subharmonic driving γ, the stationary interface position is $X = V_0\tau \sin(\Phi - \Phi_0)$, since the restoring force balances the external driving force. Fig. 3 shows the positions of the interface for various Φ. From such images we determined the equilibrium position X as function of Φ (Fig. 4a). The solid line depicts the sinusoidal fit predicted by the theory. Because of the finite mass ratio effect, the amplitude of the measured plate acceleration μ at frequency f_1 also shows periodic behavior with Φ, (see Fig. 4b), enabling us to infer the interface displacement from the acceleration measurements. For even larger amplitude of subharmonic driving (more than 4-5 % of the primary driving) extended patterns (hexagons) re-emerge throughout the cell.

The velocity V_0 which the interface would have in an infinite system, can be found from the measurements of the relaxation time τ and maximum displacement X_m at a given amplitude of subharmonic acceleration γ, $V_0 = X_m/\tau$, see Eq.(2). We verified that in the rectangular cell the displacement X depends linearly on γ almost up to values at which the interface disappears at the short side wall of the cell (see Fig. 5, inset). Figure 5 shows the susceptibility $\alpha = V_0/\gamma$ as a function of the amplitude of the primary acceleration Γ. The susceptibility decreases with Γ. The cusp-like features in the Γ-dependence of α (and τ, see inset to Fig. 2), are presumably related to the commensurability between the lateral size of the cell

and the wavelength of the interface decorations.

We developed an alternative experimental technique which allowed us to measure simultaneously the relaxation time τ and the "asymptotic" velocity V_0. This was achieved by a small detuning Δf of the additional frequency f_1 from the exact subharmonic frequency $f/2$, i.e. $\Delta f = f_1 - f/2 \ll f$. It is equivalent to the linear increase of phase shift Φ with the rate $2\pi\Delta f$. This linear growth of the phase results in a periodic motion of the interface with frequency Δf and amplitude $X_m = V_0/\sqrt{\tau^{-2} + (2\pi\Delta f)^2}$ (see Eq (2)). The measurements of the "response functions" $X_m(\Delta f)$ are presented in Fig. 6. From the dependence of X_m on Δf we can extract parameters V_0, α and τ by a fit to the theoretical function. The measurements are in very good agreement with previous independent measurements of relaxation time τ and susceptibility α. For comparison with the previous results, we indicate the values for τ and α, obtained from the response function measurements of Figs. 2 and 5 (stars). The measurements agree within 5 %.

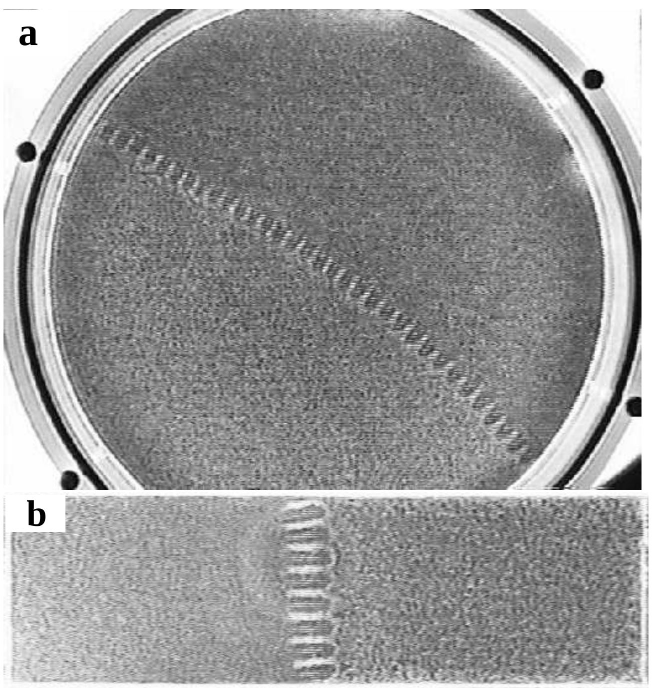

FIG. 1. Stationary position of the interface in circular, diameter 15.3 cm (a) and rectangular 4×12 cm (b) cells for sinusoidal driving force with $f = 40$ Hz and $\Gamma = 4.1$. Layer thickness 10 particle diameters.

In summary, the position of a vertically vibrated granular layer can be controlled by a very small acceleration applied at the subharmonic frequency (of the order of 0.1% of the primary harmonic acceleration). The direction and magnitude of the interface displacement depend sensitively on the relative phase of the subharmonic drive. Our measurements confirm the theoretical predictions made on the basis of the order parameter model [9]. We observed that period-doubling motion of the flat layers produces subharmonic driving because of the finite ratio of the mass of the granular layer and the cell. This in turn leads to the restoring force driving the interface towards the middle of the cell.

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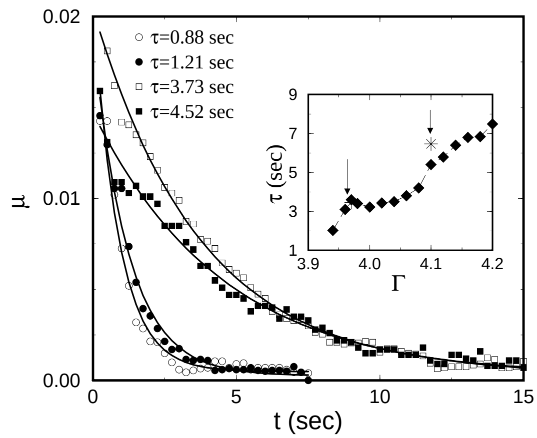

FIG. 2. Amplitude of the subharmonic acceleration μ of the cell averaged over 4 measurements vs time for the interface propagating to the center of the cell for $\Gamma = 3.97$ and $f = 40$ Hz. Circles/squares correspond to the circular/rectangular cells, open/closed symbols correspond to light/heavy cells, respectively. Heavy cells differ from light ones by an additional weight of 250 g attached to the moving shaft. Solid lines show exponential fit $\mu \sim \exp(t/\tau) + const$. Inset: τ vs Γ for light rectangular cell.

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[13] The measured acceleration μ may differ from the applied subharmonic (sinusoidal) driving γ since the granular material moves inside the cell. We measure γ independently by removing the granular material from the cell.

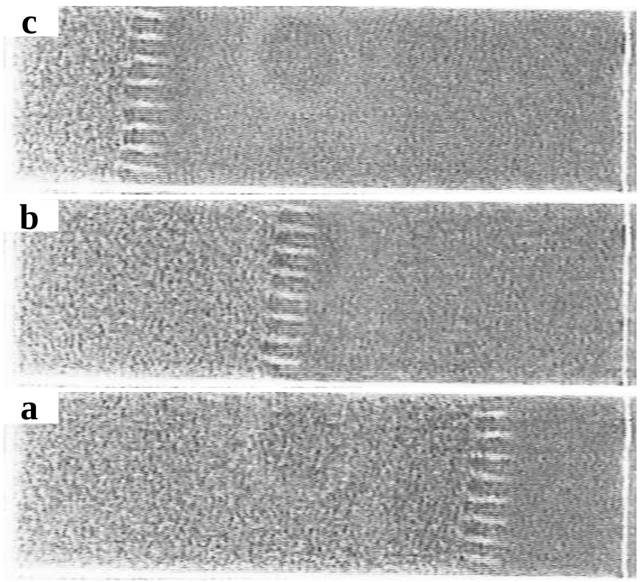

FIG. 3. Equilibrium position of interface for $\Phi = 80^0$ (a); $\Phi = 170^0$ (b); $\Phi = 260^0$ (c) for $\Gamma = 4.1$, $f = 40$Hz, $f_1 = f/2$, and $\gamma = 0.6\%$ of Γ in a rectangular cell.

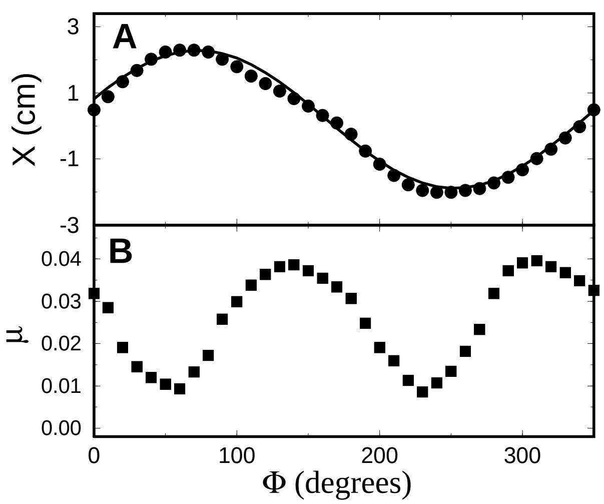

FIG. 4. (a) Equilibrium position X (b) amplitude of measured subharmonic acceleration μ as functions of phase Φ. Circular cell, $\Gamma = 4.1$, $f = 40$ Hz, $\gamma = 1.25\%$ of Γ.

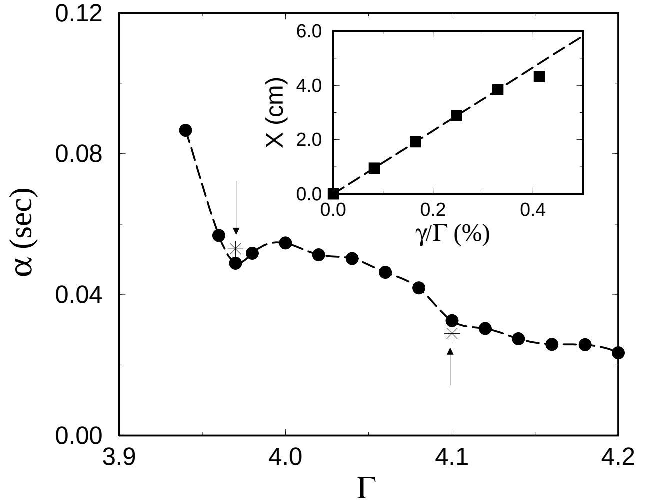

FIG. 5. Susceptibility $\alpha = V_0/\gamma$ vs Γ at $f = 40$Hz, rectangular cell. Inset: Displacement X as function of γ at $\Phi = 260^0$.

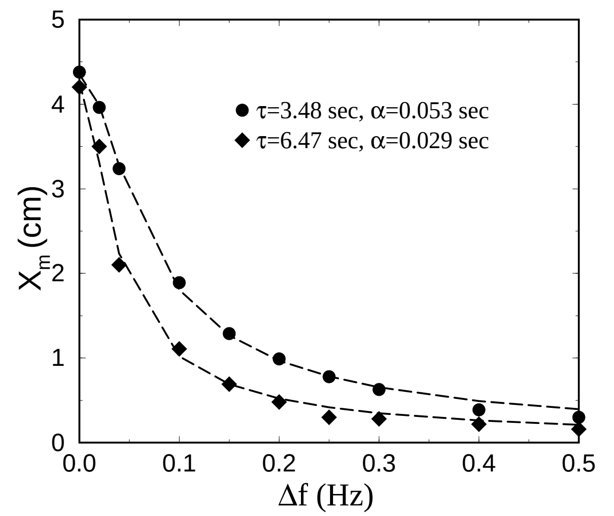

FIG. 6. Maximum displacement X_m from center of rectangular cell as function of frequency difference $\Delta f = f_1 - f/2$ for $f = 40$ Hz, and for $\Gamma = 3.97$ (circles) and $\Gamma = 4.1$ (diamonds). Dashed lines are fit to $X_m = V_0/\sqrt{\tau^{-2} + (2\pi\Delta f)^2}$. The values of α and τ obtained from the fit are also indicated in Figs. 2 and 5 (stars).